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Control of vibrations of a moving beam

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Abstract. The translational motion of a thermoelastic beam under transverse vibrations caused by initial perturbations is considered. It is assumed that a beam moving at a constant translational speed is described by a model of a thermoelastic panel supported at the edges of the considered span. The problem of optimal suppression of vibrations is formulated when applying active transverse influences to the panel. To solve the optimization problem, modern methods developed in the theory of control of systems with distributed parameters described by partial differential equations are used.

The problem of suppressing vibrations of a deformable thermoelastic system moving in the longitudinal direction is of definite theoretical and practical interest. Previously arising problems in this direction were considered in [1–3]. The problems of elastic instability of vibrations, as well as the problem of finding the critical velocity parameters of longitudinal motion [4], material temperatures, structural parameters and other properties of the moving system, which determine its critical behavior including its static and dynamic forms of stability loss, were investigated.

A new formulation of the problem of optimal suppression of transverse vibrations of a continuous panel (elastic beam) moving in the longitudinal direction is described. Under the assumption of the presence of initial perturbations of the rectilinear movement of the beam and transverse velocities, an effective algorithm for suppressing perturbations is proposed. The above algorithm is based on obtaining and using necessary optimality conditions and the arising dynamic partial differential equations. These equations describe both the vibration processes of the moving beam (direct problems) and some processes for input conjugate variables (conjugate problems). It is shown that the solution of direct and conjugate problems is possible with the Galerkin method. An example is given that illustrates the main steps in the solution of the optimal damping problem.

Let us consider a continuous thermoelastic beam (panel) moving in the longitudinal direction x with constant velocity V_0 and simply supported at points $x = \pm l$. The transverse vibrations of this panel are described by the partial differential equation [4–7]

$$m \left(\frac{\partial^2 w}{\partial t^2} + 2V_0 \frac{\partial^2 w}{\partial x \partial t} + V_0^2 \frac{\partial^2 w}{\partial x^2} \right) - \left(T_0 - \frac{Eh}{1-\nu} \varepsilon_\theta \right) \frac{\partial^2 w}{\partial x^2} + D \frac{\partial^4 w}{\partial x^4} - g(x, t) = 0, \quad (1)$$

$$(x, t) \in \Omega \{ -l \leq x \leq l, 0 \leq t \leq t_f \}$$

with boundary conditions

$$w|_{x=\pm l} = 0, \quad \frac{\partial^2 w}{\partial x^2} \Big|_{x=\pm l} = 0, \quad t \in [0, t_f] \quad (2)$$



where m is the mass per unit of panel area, $2l$ is the length of the span, h ($h \ll l$) is the panel thickness, t_s is the final time moment, $D = Eh^3/[12(1 - \nu^2)]$ is the bending rigidity (cylindrical rigidity), E is Young's modulus, and ν is Poisson's ratio. In equation (1), the expressions $\partial^2 w/\partial t^2$, $2V_0 \partial^2 w/(\partial x \partial t)$, $V_0^2 \partial^2 w/\partial x^2$ are the local acceleration, Coriolis acceleration, and centripetal acceleration, consequently. The expressions $T_0 \partial^2 w/\partial x^2$, $D \partial^4 w/\partial x^2$, $Eh\varepsilon/(1 - \nu) \partial^2 w/\partial x^2$ are the resistance force of the given tension T_0 and the elastic resistance acting against the bending and the temperature action. The generalized thermal deformation ε_θ (we suppose that the temperature is homogeneous with respect to the panel thickness) is determined as [8, 9]

$$\varepsilon_\theta = \frac{1}{h} \int_{-h/2}^{h/2} \alpha_\theta \theta dz = \alpha_\theta \theta.$$

Here h is the panel thickness, $\theta = \theta_a - \theta_0$ is the temperature discrepancy, where θ_0 is the temperature when the thermal deformations are absent, and θ_a is the actual temperature (θ , θ_a , and θ_0 are measured).

The transverse vibrations in the moving panel are due to the initial perturbations $g_1(x), g_2(x)$ of transverse displacements and its velocities, i.e., the initial conditions are determined as

$$w|_{t=0} = g_1(x), \quad \left. \frac{\partial w}{\partial t} \right|_{t=0} = g_2(x), \quad x \in [-l, l], \quad g_1 \in H^1(-l, l), \quad g_2 \in L^2(-l, l). \quad (3)$$

For vibration suppression, the transverse dynamical actions $g = g(\alpha, t)$, $g \in L^2(\Omega)$ are applied and the following isoperimetric condition is added:

$$J_\mu = \int_{-l}^l \int_0^{t_f} g^2(x, t) dt dx = M_0. \quad (4)$$

Here $M_0 > 0$ is a given constant, L^2 is the space of square-integrable functions, H^1 is the space of Sobolev functions which together with their first derivatives are square integrable.

The minimizing quality criterion is written as

$$J_g = \int_{-l}^l \left[\alpha_1 w^2 + \alpha_2 \left(\frac{\partial w}{\partial t} \right)^2 \right] \Big|_{t=t_f} dx, \quad (5)$$

where the integral J_g is considered at the final time moment t_f and α_1, α_2 are given positive constants.

The considered problem of optimization of vibrations suppression process consists in finding the control action $g(x, t)$ satisfying the isoperimetrical condition (4) and minimizing the quality functional (5) for initial boundary-value problem (1)–(3).

To obtain a necessary optimality condition, we use the equation in variations corresponding to equation (1) and the variated initial and boundary conditions (2), (3). We introduce the conjugate variable $v = v(x, t)$ and the notation $\tilde{D} = D/m$, $\tilde{g} = g/m$, $C^2 = T_0/m$, $e = Eh\theta\alpha_\theta/[(1 - \nu)m]$; the tilde is omitted in what follows. We will derive the equation determining the conjugate variable with corresponding boundary conditions and conditions at the final time moment t_f of the considered process of vibration suppression. For this purpose, let us multiply the equation in variations by $v(x, t)$ satisfying the boundary conditions in the form (2) and integrate the obtain product in the domain Ω taking into account the considered relations in variations. We obtain an expression for the functional variation δJ_a accounting for the dynamics equation.

Then, we construct the augmented functional variation $\delta J = \delta J_g + \delta J_a + \mu \delta J_\mu$, where μ is the Lagrange factor corresponding to condition (4) and

$$\begin{aligned} \delta J_g &= 2 \int_{-l}^l \left[\alpha_1 w \delta w + \alpha_2 \frac{\partial w}{\partial t} \delta \left(\frac{\partial w}{\partial t} \right) \right] \Big|_{t=t_f} dx, & \delta J_\mu &= 2 \int_{-l}^l \int_0^{t_f} g \delta g dt dx, \\ \delta J_a &= \int_{-l}^l \int_0^{t_f} \left\{ \left[\frac{\partial^2 v}{\partial t^2} + 2V_0 \frac{\partial^2 v}{\partial x \partial t} + (V_0^2 - C^2 + e) \frac{\partial^2 v}{\partial x^2} + D \frac{\partial^4 v}{\partial x^4} \right] \delta w - v \delta g \right\} dt dx \\ &+ \int_{-l}^l \left[v \delta \left(\frac{\partial w}{\partial t} \right) - \left(\frac{\partial v}{\partial t} + 2V_0 \frac{\partial v}{\partial x} \right) \delta w \right] \Big|_{t=t_f} dx. \end{aligned}$$

Taking into account that $\delta J = 0$ and $\delta g, \delta w$ are arbitrary variations for $x \in [-l, l]$, we obtain the necessary optimality condition

$$g_*(x, t) = \frac{1}{2\mu} v(x, t), \quad \mu^2 = \frac{1}{4M_0} \int_{-l}^l \int_0^{t_f} v^2 dt dx, \quad (x, t) \in \Omega,$$

the homogeneous differential equation for the conjugate variable $V(x, t)$

$$\frac{\partial^2 v}{\partial t^2} + 2V_0 \frac{\partial^2 v}{\partial x \partial t} + (V_0^2 - C^2 + e) \frac{\partial^2 v}{\partial x^2} + D \frac{\partial^4 v}{\partial x^4} = 0, \quad (x, t) \in \Omega \tag{6}$$

and the conditions at the final time moment $t = t_f (x \in [-l, l])$

$$v|_{t=t_f} = -2\alpha_2 \frac{\partial w}{\partial x} \Big|_{t=t_f}, \quad \frac{\partial v}{\partial t} \Big|_{t=t_f} = 2 \left(\alpha_1 w + 2\alpha_2 V_0 \frac{\partial^2 w}{\partial x^2} \right) \Big|_{t=t_f}. \tag{7}$$

We will use the dimensionless variable

$$\tilde{x} = \frac{x}{l}, \quad \tilde{t} = \frac{t}{\tau}, \quad \tilde{w}(\tilde{x}, \tilde{t}) = \frac{w(l\tilde{x}, \tau\tilde{t})}{h}, \quad \tilde{v}(\tilde{x}, \tilde{t}) = \frac{v(l\tilde{x}, \tau\tilde{t})}{h}, \quad \tilde{g}(\tilde{x}, \tilde{t}) \equiv \frac{lg(l\tilde{x}, \tau\tilde{t})}{mC^2} = \frac{g(l\tilde{x}, \tau\tilde{t})}{T_0}$$

and the notation

$$\alpha = \frac{l}{\tau C}, \quad \beta = \frac{D}{ml^2 C^2}, \quad v_0 = \frac{V_0}{C}, \quad \tilde{\Omega} = \{-1 \leq \tilde{x} \leq 1, 0 \leq \tilde{t} \leq \tilde{x}_f = t_f/\tau\},$$

where τ is a specific parameter with the dimension of time, and the tilde will be omitted in what follows. Then the vibration equation is

$$\alpha^2 \frac{\partial^2 w}{\partial t^2} + 2\alpha v_0 \frac{\partial^2 w}{\partial x \partial t} + (v_0^2 - 1 + e) \frac{\partial^2 w}{\partial x^2} + \beta \frac{\partial^4 w}{\partial x^4} = g(x, t). \tag{8}$$

The analogues equation for $v(x, t)$, we obtain with $g(t, x) = 0$ in (8).

For solving the nonhomogeneous partial differential equation (8) with conditions (2), (3) (the problem for determining the function w) and the corresponding homogeneous equation for v with conditions (7) and the boundary conditions in the form of (2), we apply the Galerkin method [10, 11] and represent the functions w, v in series form

$$w(x, t) = \sum_{n=1}^{n_0} f_n(t) \Psi_n(x), \quad v(x, t) = \sum_{n=1}^{n_0} q_n(t) \Psi_n(x), \quad \Psi_n(x) = \sin \left[\frac{n\pi}{2}(x+1) \right], \quad x \in [-1, 1],$$

where $\Psi_n(x)$ are shape functions satisfying the pinned simply supported boundary conditions at $x = \pm 1$, n_0 is a given number of series members, $f_n(t)$ and $g_n(t)$ ($n = 1, \dots, n_0$) are unknown time functions.

Determining the function $f_n(t)$ is reduced to integrating a system of ordinary differential equations for $f_n(t)$ with the initial conditions

$$f_j(0) = \int_{-1}^1 \Psi_j(x)g_1(x) dx, \quad \left. \frac{\partial f_j}{\partial t} \right|_{t=0} = \int_{-1}^1 \Psi_j(x)g_2(x) dx, \quad j = 1, 2, \dots$$

Determining the functions $q_n(t)$ is based on integrating the corresponding system of equations for $q_n(t)$ with the condition at the final time moment $t = t_f$

$$q_j(t_f) = -2\alpha_2 \sum_{n=1}^{n_0} \int_{-1}^1 \Psi_j(x) \frac{d\Psi_n(x)}{dx} dx f_n(t_f), \quad \left. \frac{dq_j}{dt} \right|_{t=t_f} = 2 \left[\alpha_1 - 2\alpha_2 v_0 \left(\frac{j\pi}{2} \right)^2 \right] f_j(t_f).$$

As an example, we consider the case where

$$g_1(x) = \Psi_1(x), \quad g_2(x) = 0, \quad -1 \leq x \leq 1,$$

the thermal actions are ignored, and $g = g^0(x, 1) \equiv 1$, $(x, t) \in \Omega$ (non-optimal constant actions). In this case,

$$n_0 = 1, \quad G_1(t) = \frac{4}{\pi}, \quad f_1(0) = 1, \quad \left. \frac{df_1}{dt} \right|_{t=0} = 0$$

and the equation of transverse vibrations of the moving panel becomes

$$\begin{aligned} \frac{d^2 f_1}{dt^2} + c_1 f_1 + c_2 &= 0, \quad 0 \leq t \leq t_f, \\ c_1 = \frac{\gamma_1}{\alpha^2}, \quad c_2 = \frac{G_1}{\alpha^2}, \quad \gamma_1 &= \left(\frac{\pi}{2} \right)^2 \left[- (v_0^2 - 1) + \beta \left(\frac{\pi}{2} \right)^2 \right]. \end{aligned} \tag{9}$$

Taking into account the initial conditions, we obtain the following solution

$$f_1(t) = \left(1 - \frac{c_2}{c_1} \right) \cos(\sqrt{c_1}t) + \frac{c_2}{c_1}, \quad 0 \leq t \leq t_f.$$

Then, we integrate the homogeneous equation

$$\frac{d^2 q_1}{dt^2} + c_1 q_1 = 0, \quad 0 \leq t \leq t_f, \tag{10}$$

from $t = t_f$ to $t = 0$ with the conditions

$$q_1(t_f) = -2\alpha_2 \int_{-1}^1 \Psi_1 \frac{d\Psi_1}{dx} dx f_1(t_f) = 0, \quad \left. \frac{dq_1}{dt} \right|_{t=t_f} = 2 \left[\alpha_1 - 2\alpha_2 v_0 \left(\frac{\pi}{2} \right)^2 \right] f_1(t_f) \equiv Q,$$

and obtain the solution

$$q_1(t) = Q_1 \sin(\sqrt{c_1}t) + Q_2(\sqrt{c_1}t), \quad Q_1 = \frac{Q}{\sqrt{c_1}} \cos(\sqrt{c_1}t_f), \quad Q_2 = \frac{Q}{\sqrt{c_1}} \sin(\sqrt{c_1}t_f).$$

Thus, we have approximately found the extremal loading

$$d_*(x, t) = \frac{1}{2\mu} q_1(t) \Psi_1(x), \quad \mu^2 = \frac{1}{4\mu_0} \int_{-1}^1 \int_0^{t_f} \Psi_1^2(x) q_1^2(t) dx dt.$$

For the obtained control loading, we solve equation (9), where

$$c_2 = \frac{G_1}{\alpha^2} = \frac{1}{\alpha^2} \int_{-1}^1 \Psi_1(x)g_*(x, t) dx = \frac{q_1(t)}{2\mu\alpha^2}$$

and obtain the optimal solution

$$f_1^*(t) = -\frac{Q \cos(\sqrt{c_1}t_f)}{4\mu\alpha^2 C_1^{3/2}} \sin(\sqrt{c_1}t) + \left[-\frac{Q \cos(\sqrt{c_1}t_f)}{4\mu\alpha^2 C_1^{3/2}} \right] \cos(\sqrt{c_1}t) + \frac{1}{4\mu\alpha^2 c_1} [(Q_1\sqrt{c_1}t - Q_2) \cos(\sqrt{c_1}t) - Q_2\sqrt{c_1}t \sin(\sqrt{c_1}t)].$$

In this case of optimal vibration suppression, the value of quality criterion (5) is

$$J_g^* = J_g|_{g=g^*} = \alpha_1 (f_1^*|_{t=t_f})^2 + \alpha_2 \left(\frac{df_1^*}{dt} \Big|_{t=t_f} \right)^2.$$

In the case of non-optimal action

$$g = g^0(x, t) = 1 \quad \left(c_2 = \frac{4}{\pi\alpha^2}, \frac{c_2}{c_1} = \frac{4}{\pi\alpha^2\gamma_1} \right),$$

expression (5) has the form

$$J_g^* = J_g|_{g=g^0} = \alpha \left[\left(1 - \frac{c_2}{c_1} \right)^2 \cos^2(\sqrt{c_1}t_f) + 2\frac{c_2}{c_1} \left(1 - \frac{c_2}{c_1} \right) \cos(\sqrt{c_1}t_f) + \left(\frac{c_2}{c_1} \right)^2 \right] + \alpha_2 c_1 \left(1 - \frac{c_2}{c_1} \right)^2 \sin^2(\sqrt{c_1}t_f).$$

In the case of absence of control action ($g = c_1 = c_2 = 0$), we have

$$J_g|_{g=g^0} = \alpha_1 \cos^2(\sqrt{c_1}t_f) + \alpha_2 c_1 \sin^2(\sqrt{c_1}t_f).$$

In figure 1, the dependence of functionals $J_{opt} = J_g^*(t_f)$, $J_1 = J_g^0(t_f)$ and $J_0 = J_g(t_f)|_{g=0}$ on the problem parameter t_f are presented for $\alpha_1 = \alpha_2 = 1$, $c_1 = 1$, $c_2 = 4/\pi$, $\alpha = \beta = 1$, $\gamma_1 = 1$, and $v_0 = 1.77$.

The solid line corresponds to the determined optimal control actions $g_{opt} = g_{opt}(x, t)$, the dotted line illustrates the case without control actions $g(T, t) = 0$, the dashed line characterizes the case with constant (non-optimal) actions $g(T, t) = 1$ applied to the moving beam.

Figure 1 shows that there is a time interval for which the applied optimal control actions are essentially effective.

In addition to a one-dimensional case of a moving beam, we could consider two-dimension plate models describing the longitudinal motions of wide bands in terms of tape systems. This case [4] can be based on the use of necessary conditions for the extremum and application of the Galerkin–Fourier methods too.

Conclusions

In this study a new formulation of the problem of optimal suppression of transverse vibrations of a continuous panel moving in the longitudinal direction is described.

To suppress the arising vibrations, control actions in the form $g = g(x, t)$ with non-separating independent variables x and t are used in the work. However, with this approach, the optimal control can be very complicated, and its practical implementation is difficult. In this case, it is possible to use the representation of the control action in the form $g(x, t) = f(t)\chi(x)$ with

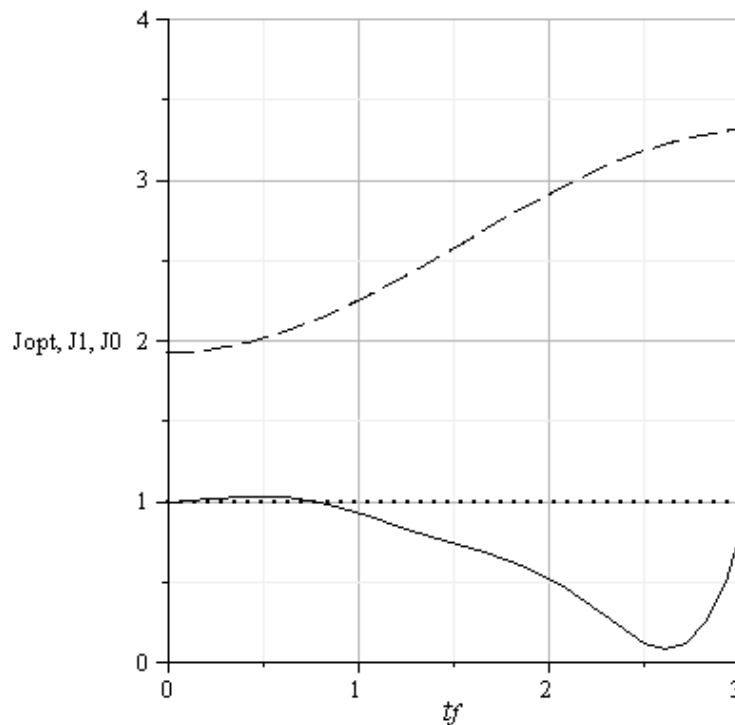


Figure 1. Dependence of the quality criterion on the problem parameter.

separated functions of position and time, describing both the specific geometric realization of location of the actions (actuators) set by $\chi(x)$ and the way of changing the time effects, denoted by $f(t)$. This approach allows us to consider and to compare the effectiveness of different methods to apply the effects to different parts of forces, and to compare such loadings of the beam applied at individual points of concentrated forces, and to compare such loadings in time as relay, harmonic, shock and other controls.

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